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Supersymmetry and the brane world

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Abstract: We investigate the possibility of gravity localization on the brane in the context of supersymmetric theories. To realize this scenario one needs to find a theory with the supersymmetric flow stable in IR at two critical points, one with positive and the other with negative values of the superpotential. We perform a general study of the supersymmetric flow equations of gauged massless supergravity interacting with arbitrary number of vector multiplets and demonstrate that localization of gravity does not occur. The same conclusion remains true when tensor multiplets are included. We analyze all recent attempts to find a BPS brane-world and conclude that localization of gravity on the brane in supersymmetric theories remains a challenging but unsolved problem.

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Contents

1. Introduction

One of the most interesting recent trends in particle physics and cosmology is the investigation of the possibility that we live on a 4-dimensional brane in a higher-dimensional universe, see e.g. [1, 2]. This is a very exciting new development, and the number of new papers on this subject grows rapidly. Some of these papers investigate phenomenological consequences of the new paradigm, some study possible changes in cosmology. However, in addition to investigating the new phenomenology it would be desirable to find a compelling theoretical realization of the new set of ideas. Here the situation remains somewhat controversial.

In this paper we will discuss a very interesting possibility, discovered by Randall and Sundrum (RS) [2]. They have found that if two domains of 5-dimensional anti-de Sitter space with the same (negative) value of vacuum energy (cosmological constant) are divided by a thin (delta-functional) domain wall, then under certain conditions the metric for such a configuration can be represented as

$$ds^2 = e^{2A(r)}dx^\mu dx^\nu \eta_{\mu\nu} + dr^2 \,. \tag{1.1}$$

Here $A(r) = -k|r|$, $k > 0$, r is the coordinate corresponding to the fifth dimension. An amazing property of this solution is that, because of the exponentially rapid decrease of the factor $e^{2A(r)} \sim e^{-k|r|}$ away from the domain wall, gravity in a certain sense becomes localized near the brane [2]. Instead of the 5d Newton law $F \sim 1/R^3$, where R is the distance along the brane, one finds the usual 4d law $F \sim 1/R^2$. Therefore, if one can ensure confinement of other fields on the brane, which is difficult, but perhaps not as difficult as confining gravity, one may obtain higher-dimensional space, which effectively looks like 4d space without the Kaluza-Klein compactification.

However, this scenario was based on the assumption of the existence of delta-functional domain walls with specific properties. It would be nice to make a step from phenomenology and obtain the domain wall configuration described in [2] as a classical solution of some supersymmetric theory. Very soon after this goal was formulated, several authors claimed that they have indeed obtained a supersymmetric realization of the RS scenario. Then some of them withdrew their statements, whereas many others continued making this claim. Some of the authors did not notice that they obtained solutions with $A(r) = +k|r|$, growing at large r, which does not lead to localization of gravity on the brane. Some others obtained the desirable regime $A(r) = -k|r|$ because of a sign error in their equations. Several authors used functions W, which they called 'superpotentials', but they have chosen functions which cannot appear in supergravity. As a result, the situation became rather confusing; the standard lore is that there are many different realizations of the RS scenario in supersymmetric theories, see e.g. a discussion of this issue in [3]. The purpose of our paper is to clarify the world-brane-supersymmetry relation, in a large class of supersymmetric theories.

The simplest supersymmetric theory in 5d space with AdS vacuum is $N = 2$ U(1) gauged supergravity [4]. The critical points of this theory have been studied in [5]. It can be naturally formulated in AdS, so one would expect that this theory is the best candidate for implementing the RS scenario. To investigate this issue it was necessary to find at least two different critical points (different AdS vacua) with equal negative values of vacuum energy, to verify their stability (which involves investigation of the sign of kinetic terms of vector and scalar fields) and to find domain wall solutions interpolating between two different stable AdS vacua.

The first important step in this direction was made by Behrndt and Cvetič [6]. They found two different AdS vacuum states where the scalar fields have correct kinetic terms, and obtained an interpolating domain wall solution. However, it was not clear whether the sign of the kinetic terms of the vector fields is correct. More importantly, vacuum energies (cosmological constants) of the two AdS solutions obtained in [6] were different, so they were not suitable for the RS scenario describing a wall surrounded by two AdS spaces with equal values of vacuum energy. Originally, Behrndt and Cvetič claimed that their domain wall solution has properties similar to those of the RS domain wall, with $A(r) \sim -k|r|$ at large $|r|$, but later they found that this was not the case: the solution was singular at $r = 0$, and $A(r)$ was growing as $+k|r|$ at large $|r|$.

Soon after that, in our paper with Shmakova [7] we found a model that admits a family of different AdS spaces with equal values of the vacuum energy and with proper signs of kinetic energy for the scalar and vector fields. At first glance, it seemed that this provided a proper setting for the realization of the RS scenario in supersymmetric theories. Indeed, we found a domain wall configuration separating two different AdS spaces with equal vacuum energies. This configuration has the metric (1.1). However, instead of $A(r) = -k|r|$, which is necessary for the localization of gravity on the wall, we have found that $A(r) \sim +k|r|$ at large $|r|$. At small $|r|$ the function $A(r)$ and the curvature tensor are singular, just as in the model of [6]. Instead of localization of gravity near $r = 0$ on a smooth domain wall, there is a naked singularity at $r \to 0$. We explained in [7] that this is a generic result which should be valid in any version of $N = 2$ U(1) gauged massless supergravity with one moduli. It was also shown in [7] that the desirable regime $A(r) = -k|r|$ is impossible not only for supersymmetric (BPS) configurations, but for any other domain wall solutions that may exist in this theory.

In this paper we will study $N = 2$ U(1) gauged massless supergravity with many moduli. To understand the possibilities to find the brane-world in such theories it is sufficient to study the behavior of the system near the critical points with the help of the supersymmetric flow equations.

This analysis will explain the main results obtained in [7], and generalize them for the theories with an arbitrary number of moduli. We will discuss the known theories with hypermultiplets and tensor multiplets included. We will also analyze the recent results obtained in other 5d supergravity theories and will show that none of the solutions that have been obtained so far lead to localization of gravity on a brane. All known examples of thick domain wall solutions with decreasing warp factor [8, 9, 10, 11] have been obtained by introducing non-supersymmetric 'superpotentials', which cannot appear in the framework of a supersymmetric theory.

We do hope that it is not really necessary to make a choice between the RS scenario and supersymmetry. However, we were unable so far to find any simple resolution of this problem. It remains a challenge to find a supersymmetric extension of the RS scenario or to prove that it does not exist.

2. BPS solutions near the critical points of massless $d = 5$, $N = 2$ gauged supergravity interacting with abelian vector multiplets

The most clear analysis of the situation can be given for $N = 2$, $d = 5$ gauged supergravity [4] interacting with an arbitrary number of vector multiplets, i.e. with arbitrary number of moduli. These theories have critical points where the moduli are constant and the metric is an AdS one.

The energy functional for static r-dependent configurations in these theories can be presented as follows [8, 10, 12]:

$$E = \frac{1}{2}\int_{-\infty}^{+\infty} dra^4 \left\{ [f_i^a(\phi^i)' \mp 3f^{ai}\partial_i W]^2 - 12\left[\frac{a'}{a} \pm W\right]^2 \right\} \pm 3\int_{-\infty}^{+\infty} dr\frac{\partial}{\partial r}[a^4 W]\,. \tag{2.1}$$

Here $\partial_i W \equiv \partial_i W/\partial\phi^i$. We have used the ansatz for the metric in the form

$$ds^2 = a^2(r)dx^\mu dx^\nu \eta_{\mu\nu} + dr^2\,, \tag{2.2}$$

where the scale factor $a(r)$ can be represented as $e^{A(r)}$. The energy, apart from the surface term, depends on one complete square with positive sign and another one with negative sign. Here the moduli ϕ^i depend on r and $W[\phi]$ is a superpotential defined in supergravity [4] via the moduli fields $h^I(\phi)$ constrained to a cubic surface $C_{IJK}h^I h^J h^K = 6$. The moduli live in a very special geometry with the metric $g_{ij}(\phi) = f_i^a f_j^b \eta_{ab}$ where f_i^a are the vielbeins of the very special geometry. The critical points of the superpotential were studied in [5], where it was also shown that they are analogous to the critical points of the central charge, defining the black hole entropy. The supersymmetric flow equations, which follow from this form of the energy functional, are given by

$$(\phi^i)' = \pm 3g^{ij}\partial_j W\,, \qquad \frac{a'}{a} = \mp W\,. \tag{2.3}$$

The warp factor of the metric decreases at large positive r under the condition that $H_r \equiv a'/a$ is negative. Note that H_r is the space analog of the Hubble constant in the FRW cosmology, where $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ and $H = \dot{a}/a$. When $H > 0$ the FRW universe is expanding, when $H < 0$ the FRW universe is collapsing. In our case, the warp factor is increasing if $H_r > 0$, and decreasing if $H_r < 0$.

It is important to stress here that the sign of the superpotential W is not a deciding factor for establishing the sign of H_r since there are two different sets of equations, either

$$\text{(i)} \quad (\phi^i)' = +3g^{ij}\partial_j W\,, \qquad \frac{a'}{a} = -W\,. \tag{2.4}$$

or

$$\text{(ii)} \quad (\phi^i)' = -3g^{ij}\partial_j W\,, \qquad \frac{a'}{a} = +W\,. \tag{2.5}$$

We are interested in critical points of the supersymmetric flow equations where all moduli take some finite fixed values ϕ^i_*

$$(\phi^i_{cr})' = 0 \qquad \Rightarrow (\partial_i W)_{cr} = 0\,. \tag{2.6}$$

As a result, the superpotential W acquires some non-vanishing constant value defining the absolute value of the 'Hubble constant' H_r at the critical point where the moduli are fixed.

$$H_r(\phi_*^i) = \left(\frac{a'}{a}\right)_{\phi_*^i} = \pm W(\phi_*^i) = \text{const}\,. \tag{2.7}$$

We have to find out whether it is possible under some condition to find a solution where a'/a is negative at the fixed points of the moduli.

It is useful to rewrite the supersymmetric flow equations (2.3) using the chain rule $a\frac{\partial}{\partial a}\phi^i = a\frac{\partial r}{\partial a}\frac{\partial \phi^i}{\partial r} = \frac{a}{a'}(\phi^i)'$. It follows that for any choice of equations (i) or (ii) we get

$$a\frac{\partial}{\partial a}\phi^i = -3g^{ij}\frac{\partial_j W}{W} \equiv \beta^i(\phi)\,. \tag{2.8}$$

This equation defines the beta functions for the supersymmetric flow equations reinterpreted as renormalization group equations. At the critical point ϕ_*^i the β-function vanishes, $\beta^i(\phi_*) = 0$. This form of equations cannot be used near the vanishing values of the superpotential W. However we will use these equations near the critical points where the moduli are fixed at finite values ϕ_*^i and $W(\phi_*) \neq 0$. To analyze the critical point we need to find out whether $\partial\beta/\partial\phi$ is positive or negative near ϕ_*.

The universality of the critical point in this class of theories follows from the very special geometry in the moduli space, which allows the calculation of the β-function near the critical point. The basic relation following from the very special geometry at the critical point[1] was derived in [5]

$$(\partial_i\partial_j W)_{cr} = \frac{2}{3}g_{ij}W_{cr}\,. \tag{2.9}$$

We find out that near the critical point

$$\frac{\partial\beta^i}{\partial\phi^j}(\phi_*) = -2\delta^i_j\,. \tag{2.10}$$

This means that near the critical point $\phi^i = \phi_*^i + \delta\phi^i$ the behavior of the moduli is

$$a\frac{\partial}{\partial a}\phi^i = (\phi^j - \phi_*^j)\frac{\partial\beta^j}{\partial\phi^i} = -2(\phi^i - \phi_*^i)\,. \tag{2.11}$$

Therefore if the system starts at some values of moduli ϕ below the fixed value ϕ_*, it will be driven to the larger values of moduli towards the fixed point. If it starts at values of moduli ϕ above ϕ_*, it will be driven back to smaller values of moduli till it will reach the fixed value. We can also solve this equation near the fixed moduli ϕ_*^i (which are a-independent):

$$\phi^i(a) = \phi_*^i + c^i a^{-2}\,. \tag{2.12}$$

[1]The analogous equation was for the first time derived in $d = 4$ supergravity in [13], which has allowed us to prove that the entropy of the supersymmetric black holes is a minimum of the BPS mass.

Here c^i are some arbitrary, undefined constants. This corresponds to a UV fixed point in quantum field theory; at large values of the scale parameter a, when $a \to +\infty$, the system is driven towards the fixed point. It follows that the stable critical point of moduli requires that *the scale factor of the metric in the direction towards the fixed point grows.*

As we see, eq. (2.12) is completely universal, does not depend on the number of moduli, choice of the cubic surface, etc. It also shows that the warp factor at the critical points, where the moduli are fixed, is always increasing as the 'Hubble constant" H_r is positive. We have thus established that in $d = 5$, $N = 2$ U(1)-gauged massless supergravities [4] with an arbitrary number of vector multiplets and any choice of the cubic surface, the 'Hubble constant' H_r is *always positive* at the supersymmetric critical points, where all scalars are fixed, and the warp factor is therefore *always increasing* at $|r| \to +\infty$:

$$H_r(\phi_*) = \left(\frac{a'}{a}\right)_{\phi_*} = |W_{cr}| > 0\,. \tag{2.13}$$

This is a prediction that, in all particular cases, if one starts solving the system of equations (2.3), for each sign of W, the consistent solution for $r > 0$ will be only possible in case (i) for $W < 0$ and in case (ii) for $W > 0$. In one moduli case where the relevant solutions were presented in [6] and [7] this was indeed the case: it was impossible to get the decreasing warp factor in BPS domain walls of massless gauged $d = 5$ $N = 2$ supergravity.

Here we have shown (see also [7]) that the deep reason for this is related to the fact that the second derivative of the superpotential must have the same sign as the superpotential at the fixed point of moduli, as shown in eq. (2.9). This leads to the UV fixed point universally for this class of theories.

3. Multivalued nature of the superpotential in $N = 2$ supergravity

The investigation performed in the previous section is sufficient to show, even without a detailed study of the structure of domain walls in $d = 5$, $N = 2$ supergravity, that BPS states with gravity localization do not appear in this theory. However, it is still interesting to find out exactly what kind of vacua and interpolating solutions are possible in this theory.

The existence of the disconnected branches of the moduli space (different AdS vacua) in $d = 5$, $N = 2$ supergravity interacting with vector multiplets was discovered in [14] and studied more recently in [6], [7] and [12]. The existence of the disconnected branches of the moduli space leads to a possibility to use two different branches of the moduli space and therefore two different superpotentials on each side of the wall.

This property allows in principle a solution to be found for the scalars interpolating between two branches of the moduli space. However, the space-time metric has naked singularities and the warp factor always increases away from the wall.

Here we will explain first of all why different branches of moduli space are quite natural in $d = 5$, $N = 2$ supergravity interacting with vector multiplets. The 'kinky supergravity' of Günaydin, Sierra and Townsend [14] has the following set up. The theory is defined by a cubic form

$$\frac{1}{6} C_{IJK} h^I h^J h^K = F \,, \qquad F = 1 \,, \tag{3.1}$$

and a linear form

$$W = h^I V_I \,. \tag{3.2}$$

The constants C_{IJK} define the cubic surface and the constants V_I define the superpotential. The independent scalar fields ϕ^i are coordinates on the cubic hypersurface F. The restrictions on C_{IJK} is such that the metric defining the coupling of vector fields $a_{IJ} \sim -\partial_I \partial_J F$ for $F > 0$ and the metric, induced on the surface, $g_{ij} = a_{IJ} h^I_{,i} h^J_{,j}$ are positive definite. There exists one point at the surface where a_{IJ} is simply an Euclidean metric δ_{IJ}. This is a so called 'basepoint'. The existence of the 'basepoint' has allowed the authors of ref. [4] to prove that in the one-modulus case the following constants can be chosen:

$$C_{000} = 1 \,, \qquad C_{011} = -\frac{1}{2} \,, \qquad C_{001} = 0 \,, \qquad C_{111} = C \,. \tag{3.3}$$

The most general case of arbitrary C_{IJK} for $I = 1, 2$ can be reduced to this one. The cubic polynomial takes the following form (for $\phi = h^0/h^1$)

$$F \sim (h^1)^3 \left(\phi^3 - \frac{3}{2}\phi + C \right) . \tag{3.4}$$

The discriminant of the cubic polynomial

$$\Delta = C^2 - \frac{1}{2} \tag{3.5}$$

can be positive, zero or negative. In these three cases one has 1, 2 or 3 branches of the moduli space, respectively, i.e. the curves $F = 1$ have 1, 2 or 3 branches. In this form the metric of the moduli space is known only at the 'basepoint'. Thus, a priori one may have expected that some of the branches can be excluded if the metric is not positive there.

In more recent studies of the branches of the moduli space related to domain walls a somewhat different set up was taken. The main emphasis was to find the disconnected branches of the moduli space such that the scalar and the vector metric are positive-definite; between branches the metric may be infinite, however. The expression for the moduli space metric is [12]

$$g_{\phi\phi} = \alpha \left[(C_{100}^2 - C_{110}C_{000})\phi^2 - (C_{111}C_{000} - C_{110}C_{001})\phi + C_{110}^2 - C_{000}C_{100} \right], \quad (3.6)$$

where $\alpha > 0$. For the case (3.3) we get

$$g_{\phi\phi} = \frac{1}{2}\alpha \left(\phi^2 - 2C\phi + \frac{1}{2} \right). \quad (3.7)$$

The condition that the metric is everywhere positive is that

$$\Delta = C^2 - \frac{1}{2} < 0. \quad (3.8)$$

When we combine the information from the two approaches, in [14] and in [7, 12], we find that the case with 3 branches of the moduli space defined in [14] automatically leads to the positive metric, whereas in cases when $\Delta > 0$ or $\Delta = 0$ there are parts of the moduli space where the metric is not positive. To have positive and negative superpotentials in different branches of the moduli space turns out to be a necessary condition for the positivity of the vector space metric [7]. Thus in $d = 5$, $N = 2$ supergravity interacting with vector multiplets with the same values of C_{IJK} and V_I it is possible to find two distinct stable AdS critical points.

The stability of different AdS vacua is a necessary but not a sufficient condition for the realization of the RS scenario. One must find values of the parameters C_{IJK} and V_I that allow the existence of different stable AdS vacua with equal vacuum energy density. This problem is rather non-trivial, but fortunately one can find several continuous families of parameters that satisfy this condition [7]. Once this problem is solved, one may try to find an interpolating domain wall solution separating two different stable AdS vacua with equal values of the vacuum energy.

One such solution of eqs. (2.5) was found in [7]; it is represented here in figure 1. As we see, the scalar field grows from its negative critical value $\phi = -0.2$ at $r \to -\infty$ to a positive value $\phi = 1$ at $r \to +\infty$. The superpotential discontinuously changes its sign from negative to positive at $r = 0$. At large r the function $A(r)$ grows as $|r|$ rather than decreases as $-|r|$. Thus, just as we expected, there is no gravity localization in this scenario.

Even though the solution for the scalar field ϕ smoothly interpolates between the two attractor solutions, the function $A(r)$ is singular. It behaves as $\log |r|$ at $|r| \to 0$. Metric near the domain wall is given by

$$ds^2 = r^2 dx^\mu dx^\nu \eta_{\mu\nu} + dr^2. \quad (3.9)$$

This implies the existence of a naked singularity at $r = 0$, which separates the universe into two parts corresponding to the two different attractors.

In the same theory there is also a second BPS solution, which corresponds to the different choice of sign of the pair of equations, as in eq. (2.4). The solution is shown in figure 2. This configuration has a negative superpotential at large positive r and decreasing field ϕ, but both solutions have the same warp factor. This illustrates the statement made in the previous section that the flow equation and the resulting geometry of BPS states does not depend on the choice between the two equations (2.4) and (2.5).

We conclude that one can find more than one stable AdS critical point in $N = 2$ $d = 5$ supergravity. However, this does not lead to localization of gravity on the domain wall separating two different AdS vacua.

4. Non-BPS solutions near the critical points of massless gauged supergravity

Until now we were looking only for supersymmetric solutions, and found that they do not have the desirable behavior $a \to 0$ at the points where scalars are stabilized (e.g. at $|r| \to \infty$). One may wonder whether one can find more general, non-supersymmetric solutions with the asymptotic $a \to 0$. The answer to this question is also negative. Indeed, the relevant equation of motion for the scalars in the background metric is

$$(\phi^i)'' + 4H_r(\phi^i)' + g^{ij}g_{jl,k}(\phi^k)'(\phi^l)' - 6g^{ij}\partial_j V = 0\,, \tag{4.1}$$

where at the critical points $V_{,ij}$ is negative-definite. The potential is defined as $V = -6(W^2 - (3/4)g^{ij}W_iW_j)$. For all massless $d = 5$, $N = 2$ gauged supergravities under discussion the second derivative of the potential is proportional to the potential, that is negative at the critical point:

$$(\partial_i\partial_j V)_{cr} = \frac{2}{3}g_{ij}V_{cr}\,. \tag{4.2}$$

Let us assume that the solution of this equation asymptotically approaches an attractor point ϕ_* at large $r > 0$, so that $g_{ij}(\phi_*)$ and $g_{ij,k}(\phi_*)$ become constant, and $(\phi^i)'$ gradually vanishes at large r. We will assume that H_r is negative near the critical point since we are looking for solutions with decreasing warp factor. Then the deviation $\delta\phi^i$ of the field ϕ^i from its asymptotic value ϕ^i_* at large positive r satisfies the following equation:

$$(\delta\phi^i)'' - 4|H_r|(\delta\phi^i)' = -4|V|\delta\phi^i\,. \tag{4.3}$$

Thus for each scalar field we have the same equation as for a harmonic oscillator with a negative friction term $-|A'|\delta\phi'$. Solutions of this equation describe oscillations of $\delta\phi$ with the amplitude blowing up at large $r > 0$, which contradicts our assumptions.

Let us explain this argument in a more detailed way. We are looking for asymptotic solutions of this equation at large r. In this limit all parameters of this equation take some constant values: $2|H_r| = C_1$, $4|V| = C_2$, where $C_i > 0$. Then eq. (4.3) reads:

$$(\delta\phi^i)'' - 2C_1(\delta\phi^i)' + C_2\delta\phi^i = 0\,. \tag{4.4}$$

Solutions of this equation can be represented as $\delta\phi^i = e^{i\omega r}$, where ω may take complex values. Then this equation implies that

$$\omega^2 + i2C_1\omega - C_2 = 0\,, \tag{4.5}$$

which yields

$$\omega = -i\left(C_1 \pm \sqrt{C_1^2 - C_2}\right), \tag{4.6}$$

and

$$\delta\phi^i = e^{i\omega r} = \exp\left[\left(C_1 \pm \sqrt{C_1^2 - C_2}\right) r\right]. \tag{4.7}$$

Thus at large r eq. (4.3) has two independent solutions. Both solutions grow exponentially in the limit $r \to \infty$. This means that our assumption that the solution can asymptotically approach a constant value is incompatible with the condition that $H_r < 0$. In this proof it was essential also that at the critical points $V_{,ij}$ is negative-definite (which means that the curvature of the effective potential is negative). Indeed, for $V_{,ij} > 0$ one would have $C_2 < 0$, and one of the two solutions given in eq. (4.7) would exponentially decrease at infinity, which is the required regime. But this regime is impossible in massless U(1)-gauged supergravity where $V_{,ij}$ is always negative near the attractor.

5. Solutions of other supergravity theories with AdS critical points

Here we will give a short overview of the possibilities.

1. We start with a comment on $d = 5$, $N = 8$ gauged supergravities. For these theories one finds out that, in known cases of supersymmetric flow equations presented in the literature [15, 16, 17], the first order BPS-type equations have the form $H_r = -\frac{1}{3}W$ and the superpotential at all known critical points is negative. Some of these critical points with maximal unbroken supersymmetry have a UV fixed point behavior, some have saddle points with smaller supersymmetry unbroken and have a IR point behavior. However since the superpotential is negative at all known critical points one cannot realize the situation that H_r is positive at $r \to +\infty$, which would correspond to a decreasing warp factor away from the wall in the positive r direction. No such theory seems to be available in the literature.

2. A version of 5d supergravity interacting with the vector multiplets and the so-called universal hypermultiplet was found in [18]. If the hypermultiplet is gauged, there is a contribution to the potential, which does not allow an AdS vacuum in this theory. If the gauging of the universal hypermultiplets is removed, the AdS critical points are possible, however, they are defined by the vector multiplets exclusively. The problem is reduced to the one that was studied before and there is no world-brane BPS walls with decreasing warp factor.

3. The recently discovered $N = 2$, $d = 5$ gauged supergravity with vector and tensor multiplets [19] has a new type of a potential:

$$V = 2g^2 W^{\tilde{a}} W^{\tilde{a}} + g_R^2(-P_0^2 + P^{\tilde{a}} P^{\tilde{a}})\,. \tag{5.1}$$

The scalars from vector multiplets give the usual contribution, proportional to g_R^2; here P_0 is a superpotential and $P^{\tilde{a}}$ is proportional to the derivative of the superpotential over the moduli. The new potential has an additional contribution, proportional to g^2, due to tensor multiplets, which is manifestly non-negative. The BPS form of the action consists of 3 full squares, i.e. in addition to all terms in eq. (2.1) there is a positive contribution to the energy $2g^2 W^{\tilde{a}} W^{\tilde{a}}$.

To understand the situation near the critical points in this class of theories, consider the supersymmetry transformations (with vanishing fermions)

$$\begin{aligned} \delta\psi_\mu^i &= \nabla_\mu \epsilon^i + \frac{i}{2\sqrt{6}} g_R P_0(\phi)\Gamma_\mu \delta^{ij}\epsilon_j\,, \\ \delta\lambda^{i\tilde{a}} &= -\frac{i}{2} f_{\tilde{x}}^{\tilde{a}} \Gamma^\mu (\mathcal{D}_\mu \phi^{\tilde{x}})\epsilon^i + gW^{\tilde{a}}\epsilon^i + \frac{1}{\sqrt{2}} g_R P^{\tilde{a}}(\phi)\delta^{ij}\epsilon_j\,. \end{aligned} \tag{5.2}$$

At the critical point where the moduli are constant the unbroken supersymmetry requires that

$$\delta\lambda^{i\tilde{a}} = gW^{\tilde{a}}\epsilon^i + \frac{1}{\sqrt{2}} g_R P^{\tilde{a}}(\phi)\delta^{ij}\epsilon_j = 0\,. \tag{5.3}$$

If the full $N = 2$ supersymmetry is unbroken at the critical point, we have to require that.[2]

$$gW^{\tilde{a}}(\phi_*) = \frac{1}{\sqrt{2}} g_R P^{\tilde{a}}(\phi_*) = 0\,. \tag{5.4}$$

Without tensor multiplets near the critical point where the scalars are not fixed, the r-derivative of scalars is proportional to the derivative of the superpotential

[2]We have learned that M. Günaydin has found the same condition for the supersymmetric fixed points in this theory (private communication).

and only 1/2 of supersymmetry is unbroken. In presence of tensor multiplets we may try to relax the condition for the critical point and request that $gW^{\tilde{a}}(\phi) \neq 0$. One can verify that this is not possible if any supersymmetry is unbroken. Consider a condition that all 3 bosonic terms in the gaugino supersymmetry transformation are not vanishing:

$$\phi' \sim W^{\phi} \sim P^{\phi} \,. \tag{5.5}$$

We have to find a projector specifying the Killing spinor. Under such condition the Killing spinors in addition to the usual constraint $i\Gamma^r \epsilon^i = \pm\delta^{ij}\epsilon_j$ have to satisfy the following condition:

$$\epsilon^i = \epsilon^{ij}\epsilon_j = \pm\delta^{ij}\epsilon_j \,. \tag{5.6}$$

It can be verified that this is possible only if $\epsilon_i = 0$, which means that supersymmetry is completely broken. However if we request that

$$\phi' \sim P^{\phi} \,, \qquad W^{\phi} = 0 \tag{5.7}$$

we find the usual Killing spinor projector for 1/2 of unbroken supersymmetry which is also consistent with the gravitino transformation. Also the gravitino transformation rules have integrability condition for the existence of Killing spinors; this tells us, as in the case of vector multiplets only, that the AdS curvature is defined by the value of the superpotential $P_0^2(\phi_*)$.

This brings us back to the previously studied situation with vector multiplets only, where we know that $H_r > 0$ at positive r. Therefore we conclude that the supersymmetric critical points of $N = 2$, $d = 5$ gauged supergravity with vector and tensor multiplets [19] have the same nature as the ones without tensor multiplets and therefore will not support the supersymmetric brane-world scenario.

The non-supersymmetric solutions in this theory require an additional investigation.

4. Dilatonic domain walls were studied by Youm [20] in the context of the brane-world scenario. It was found that the warp factor in the spacetime metric increases as one moves away from the domain wall for all the supersymmetric dilatonic domain wall solutions obtained from the (intersecting) BPS branes in string theories through toroidal compactifications.

5. An interesting development was pursued recently in the framework of the massive gauged supergravity in [21]. A short summary of the situation is the following. The model has one AdS critical point with the fixed scalars. It has

an IR behavior since at the critical point, in notation of [21], the β-function in the supersymmetric flow equations has the opposite sign from that of W and

$$(\partial_i \partial_j W)_{cr} = -\frac{2}{3} W_{cr}\,, \tag{5.8}$$

and therefore one finds that $\phi = \phi_* + a^4$. Since W is negative at this critical point, it gives a decreasing warp factor at $r \to -\infty$. The second AdS critical point is absent: there is only a run-away dilaton behavior. Therefore this solution does not lead to the localization of gravity.

6. Non-supersymmetric choice of the 'superpotential'

In addition to supersymmetric theories, one may consider non-supersymmetric theories with potentials that can be represented in a form $V = -\frac{1}{3}W^2 + \frac{C}{8}W_\phi^2$. This resembles potentials in supersymmetric theories with superpotential W, where the constant C depends on the choice of the theory [8, 9, 10, 11]. Then one may choose the 'superpotential' W in a way that the brane world scenario has the desirable solution with decreasing warp factor away from the wall in both directions. As follows from our analysis, one should find in the examples of this kind two IR critical points with the opposite signs of the 'superpotential'. This means that on the wall the 'superpotential' must vanish. The second derivative of the 'superpotential' must be positive (negative) when the 'superpotential' is negative (positive). This is indeed the property of the solutions found in [10, 11]. For example, in [11]

$$W(\phi) = 3 \sin \sqrt{\frac{2}{3}}\phi\,, \qquad \phi(r) = \sqrt{6} \arctan\left(\tanh \frac{r}{2}\right), \qquad a(r) = e^A = \frac{1}{2\cosh r}\,. \tag{6.1}$$

At the right critical point at $r \to +\infty$, W is positive but $W_{\phi\phi}$ is negative. At the left critical point at $r \to -\infty$, W is negative but $W_{\phi\phi}$ is positive. Therefore at both critical points (with $H_r = -W/3$ and $\phi' = W_{,\phi}/2$) one has

$$(\partial_\phi \partial_\phi W)_{cr} = -\frac{2}{3} W_{cr}\,, \quad \Rightarrow \quad \phi - \phi_* \sim a\,. \tag{6.2}$$

At both critical points, $a \to 0$ is a stable point where the scalar field reaches a fixed value. At $r = 0$ the 'superpotential' vanishes, so that to the right from the wall it is positive and to the left it is negative.

The solution for the 'superpotential' proposed in [10] has the same basic features as the one in [11]

$$W(\phi) = 2\left(\phi - \frac{1}{3}\phi^3\right), \qquad \phi(r) = \tanh r\,. \tag{6.3}$$

$$(\partial_\phi \partial_\phi W)_{cr} = -3W_{cr}\,, \qquad \Rightarrow \qquad \phi - \phi_* \sim a^{9/2}\,. \tag{6.4}$$

We plot this solution on figure 3. Note that the function $A(r)$ has a desirable behavior at large $|r|$.

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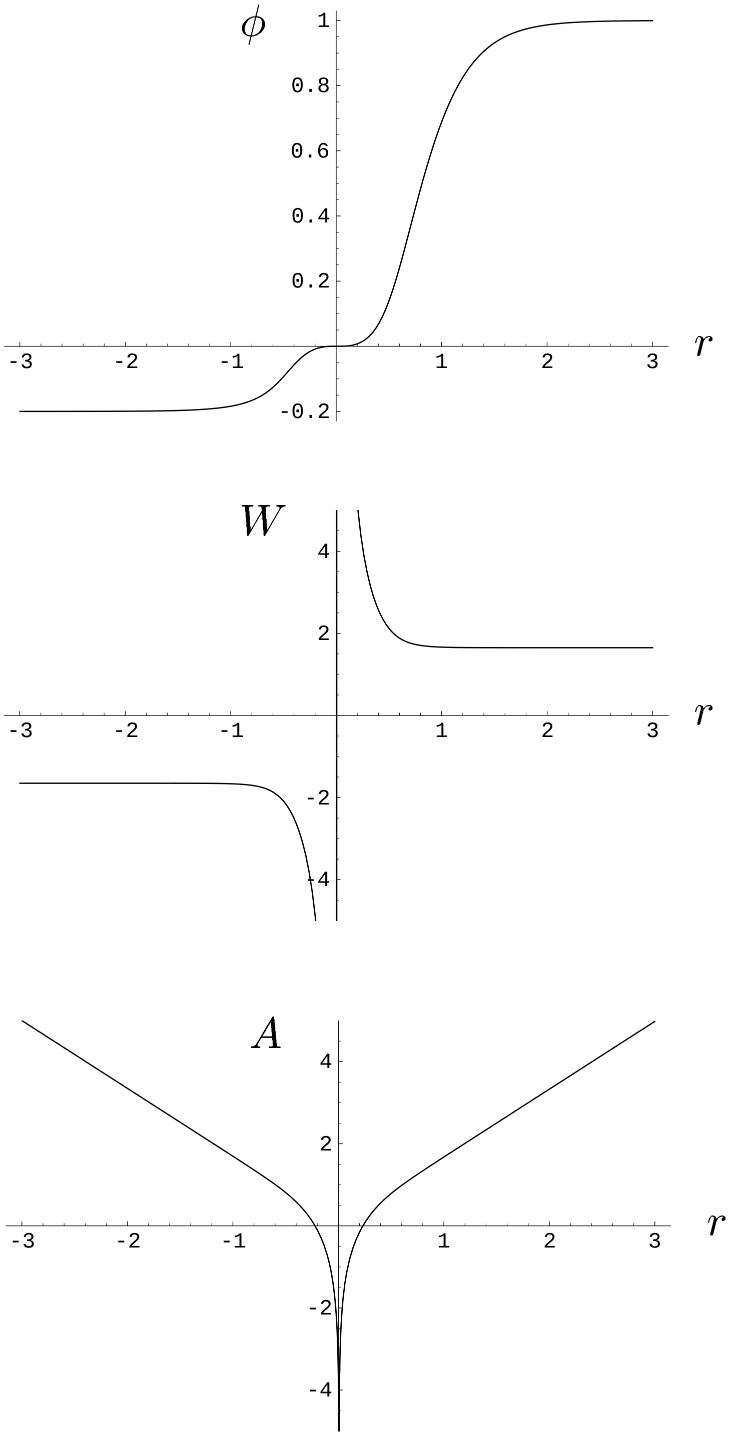

Figure 1: A solution for the scalar field ϕ interpolating between two different vacua with equal values of $|W|$ [7]. Note that $\phi(r)$ is non-singular on the wall because of the vanishing of g^{-1} at $\phi = 0$, whereas W and A are singular: $W \sim r^{-1}$ and $A(r) \sim \log|r|$ at $|r| \to 0$. At large r the function $A(r)$ grows as $|r|$ rather than decreases as $-|r|$. This is a general property of interpolating solutions in our class of models.

It may be useful to compare these solutions with the BPS ones in figures 1 and 2. One observes that in the non-supersymmetric case in figure 3 the 'superpotential' vanishes at $r = 0$ and changes its sign there. Meanwhile in the supersymmetric case, figure 1 and 2, the true superpotential changes sign on the wall, but it goes through a discontinuity.

Thus, in those cases where the brane-world scenario can be realized the 'superpotential' has the following basic features (with the choice of flow equations $H_r = -W/3$ and $\phi' = W_{,\phi}/2$):

$$W_{r=0} = 0\,, \tag{6.5}$$

and also

$$\begin{aligned} W_{r\to-\infty} &< 0\,, & W_{r\to+\infty} &> 0\,, \\ (\partial_\phi^2 W)_{r\to-\infty} &> 0\,, & (\partial_\phi^2 W)_{r\to+\infty} &< 0\,. \end{aligned} \tag{6.6}$$

Note that in both cases discussed above, at both critical points, $(\partial_\phi\partial_\phi W)_{cr}$ and W_{cr} have opposite signs, which is impossible in massless gauged supergravity. No supersymmetric embedding have been found for such 'superpotentials' so far. Thus the use of the word 'superpotential' is not quite appropriate here since it makes an incorrect impression that the theory with the 'superpotentials' described above is supersymmetric.

The study of non-linear perturbations around such non-supersymmetric solutions was performed in [9], where it was found that some 'pp curvature' singularities appear at large r. Interestingly, these singularities at large r do not appear when the proper supersymmetric superpotentials are used. A closely related singularity at the AdS horizon was discussed in [22], where the study of the black holes on domain walls was performed. These issues require further investigation.

Another problem is related to quantum effects. Usually, after taking into account one-loop corrections, the effective potential in non-supersymmetric theories cannot be represented in the form $V = -\frac{1}{3}W^2 + \frac{C}{8}W_\phi^2$. Therefore the notion of 'superpotential' becomes irrelevant and, instead of solving first-order equations for BPS-type states, one should investigate solutions of the usual second-order Lagrange equations.

In conclusion we would like to point out that the analysis performed here shows that the brane world is not yet realized as a BPS or non-BPS configuration of supersymmetric theory. We cannot exclude, however, that some supersymmetric theory can be found where such brane world may exist, providing a consistent alternative to compactification. As the present investigation shows, it may be rather non-trivial to find such a theory, if it exists, since the most general 5-dimensional supergravity theories have not been constructed yet. Since the main result depends on the specific sign of the beta function (in two critical points), one would not like to miss the existence of the correct theory. It was non-trivial to replace the positive β-function in QED by a negative β-function in non-abelian gauge theories. We hope that the analysis performed here will help to make a final conclusion on the compatibility of supersymmetry with the brane world scenario.

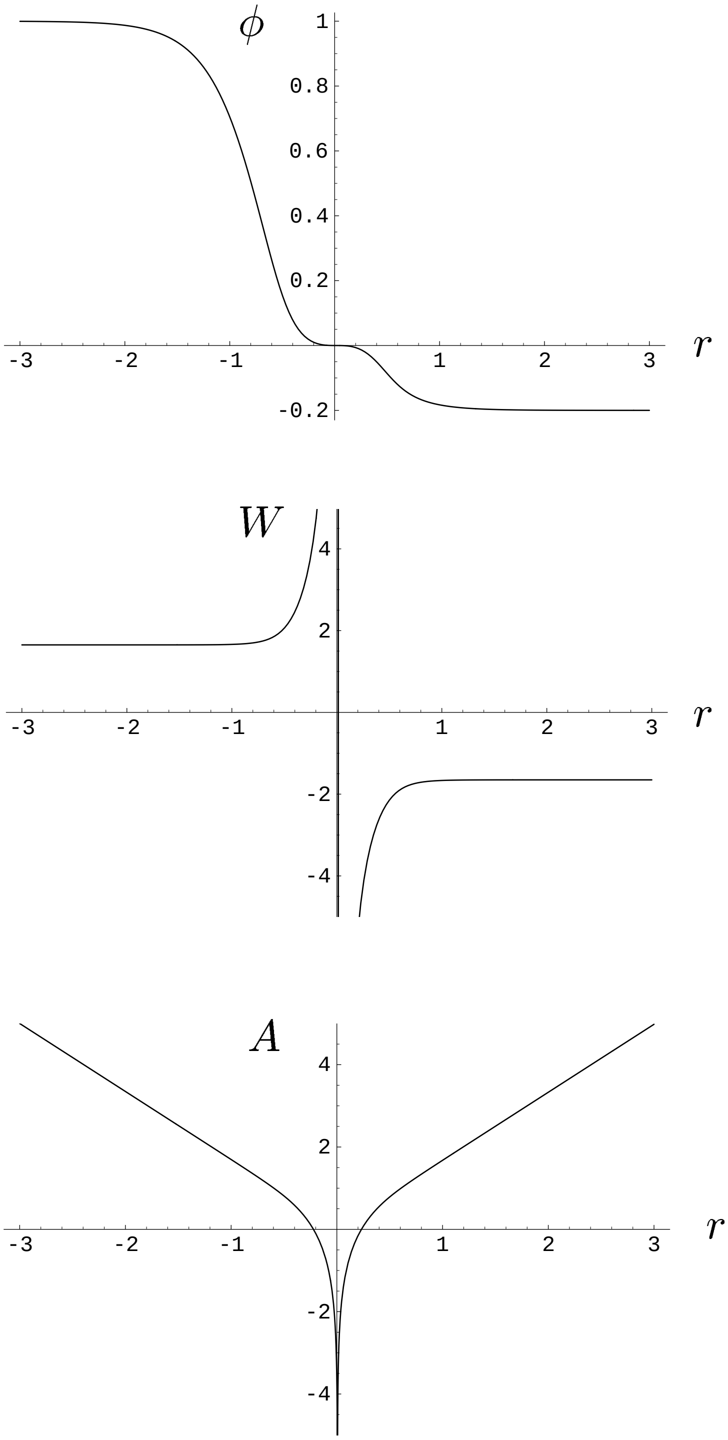

Figure 2: This solution differs from the one in figure 1 by the sign of the superpotential. Therefore the scalar field interpolates between 1 on the left and -0.2 on the right whereas in figure 1 it interpolates between -0.2 on the left and 1 on the right. However, the warp factor increases at large $|r|$ in both cases.

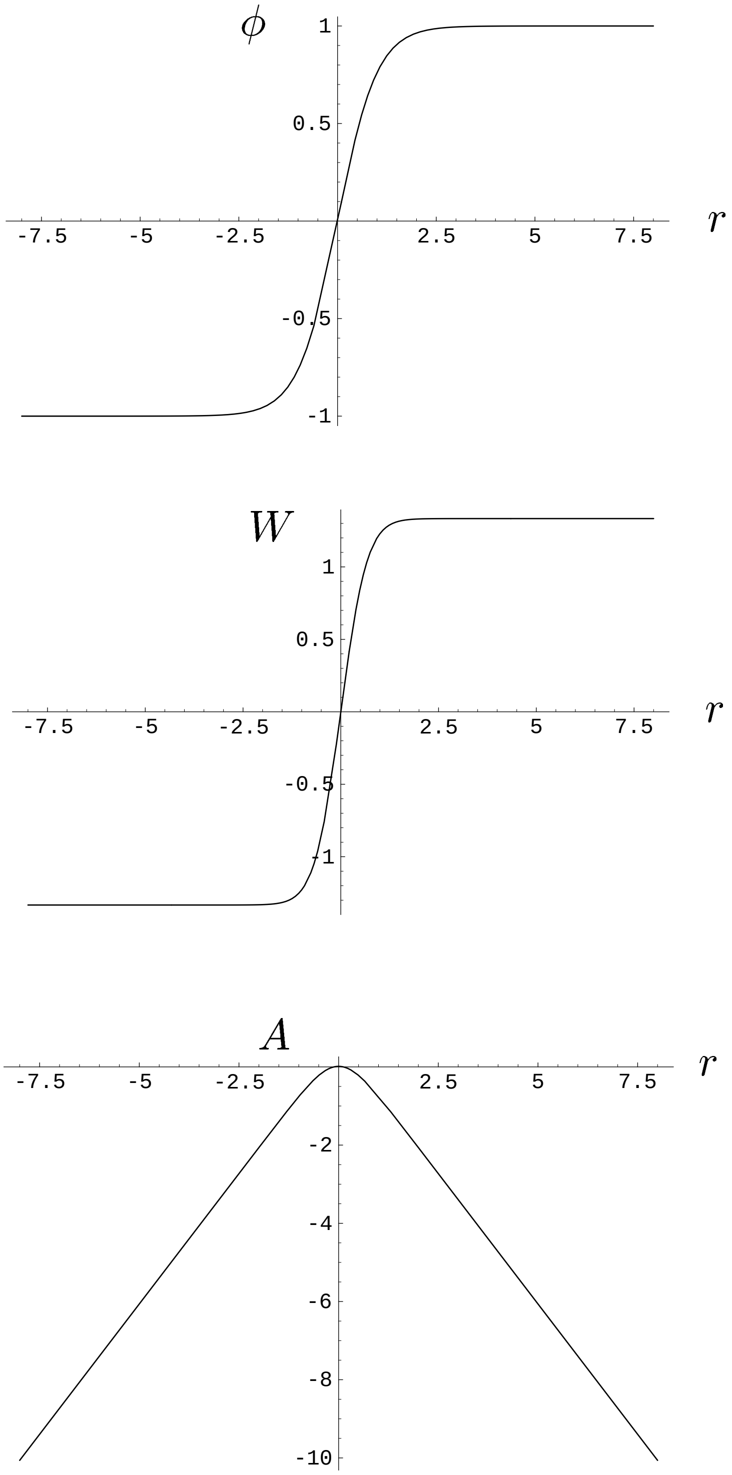

Figure 3: A domain wall solution for the scalar field ϕ in the theory with the 'superpotential' $W(\phi) = 2(\phi - \frac{1}{3}\phi^3)$ [10]. At large r the function $A(r) \sim -|r|$, just as required for gravity localization. However, the theory with this 'superpotential' is not supersymmetric.

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